PERTH MODERN SCHOOL

UNIT 3C/3D MAS – 2012

INVESTIGATION 3 – MATRICES

PART 2 – IN-CLASS VALIDATION

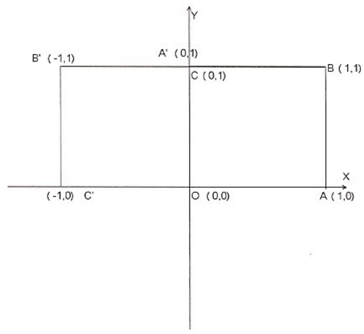
NAME: *SOLUTIONS – Part 1 + Part 2 = 66 marks* **DATE:** _______________

[To achieve full marks, working and reasoning should be shown.]
[A maximum of 2 marks will be deducted for incorrect rounding, units, notation, etc.]

Time allowed: 55 minutes ***Marks: 40***

In this section we are going to complete various rotations on the unit square. By considering the image points A′ and C′ we should be able to identify the transformation matrix R. We also want to generalise the term for the rotation matrix.

Question 8 [1, 1, 1, 2 = 5 marks]
Suppose matrix R_{90} performs a rotation 90^0 anti-clockwise about the origin (0, 0) on the unit square M.



a) Complete this transformation in the diagram opposite.

✓[Draws the image figure correctly]

b) Fill in the results to the matrix operation.

$T_1 \times M = M'$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{array}{c} \begin{matrix} O & A & B & C \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \begin{array}{c} \begin{matrix} O & A' & B' & C' \end{matrix} \\ \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

✓[Completes the image matrix from the diagram]

c) Hence complete the following:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

✓[Defines the transformation matrix]

d) Complete the trig ratios
$\sin 90° = \mathbf{1}$ $\cos 90° = \mathbf{0}$

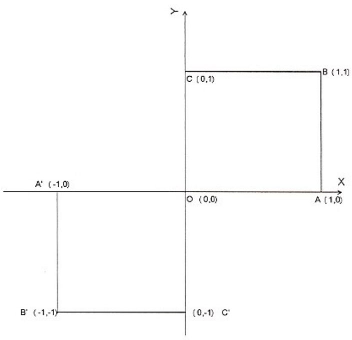
and substitute the trig terms in place of the zeros and ones in the transformation matrix.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{bmatrix}$$

✓[Defines the trig values]

✓[Replaces them in the matrix]

Question 9 [1, 1, 1, 2 = 5 marks]
Suppose matrix R_{180} performs a rotation 180^0 anti-clockwise about the origin (0, 0) on the unit square M.



a) Complete this transformation in the diagram opposite.

✓[Draws the image figure correctly]

b) Fill in the results to the matrix operation.

$R_{180} \times M = M'$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \left[\begin{array}{cccc} O & A & B & C \\ \hline 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right] = \left[\begin{array}{cccc} O & A' & B' & C' \\ \hline 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & -1 \end{array}\right]$$

✓[Completes the image matrix from the diagram]

c) Hence complete the following:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

✓[Defines the transformation matrix]

d) Complete the trig ratios
$\sin 180° = \boldsymbol{\mathit{0}}$ $\cos 180° = \boldsymbol{\mathit{-1}}$

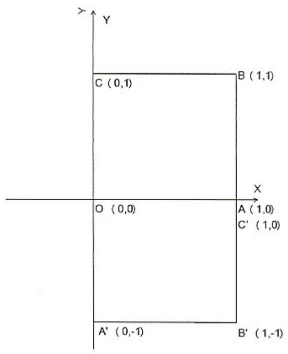
and substitute the trig terms in place of the zeros and ones in the transformation matrix.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \cos 180 & -\sin 180 \\ \sin 180 & \cos 180 \end{bmatrix}$$

✓[Defines the trig values]

✓[Replaces them in the matrix]

Question 10 [1, 1, 1, 2 = 5 marks]



Suppose matrix T_7 performs a rotation 270^0 anti-clockwise about the origin (0, 0) on the unit square M.

a) Complete this transformation in the diagram opposite.

✓[Draws the image figure correctly]

b) Fill in the results to the matrix operation.

$R_{270} \times M = M'$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{array}{c} \begin{array}{cccc} O & A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} O & A' & B' & C' \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & -1 & -1 & 0 \end{bmatrix} \end{array}$$

✓[Completes the image matrix from the diagram]

c) Hence complete the following:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

✓[Defines the transformation matrix]

d) Complete the trig ratios

$\sin 270° = \boldsymbol{-1}$ $\qquad\qquad$ $\cos 270° = \boldsymbol{0}$

and substitute the trig terms in place of the zeros and ones in the transformation matrix.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \cos 270 & -\sin 270 \\ \sin 270 & \cos 270 \end{bmatrix}$$

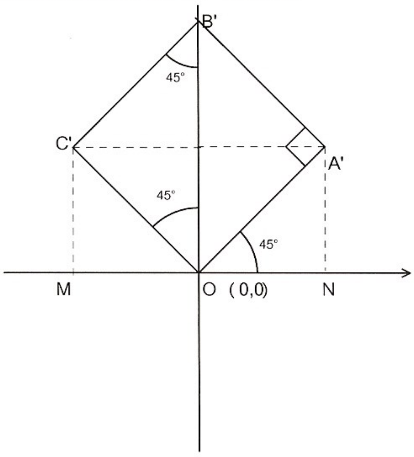
✓[Defines the trig values]

✓[Replaces them in the matrix]

Question 11 [5, 2, 1, 2 = 10 marks]

Suppose matrix R_{45} performs a rotation 45^0 anti-clockwise about the origin (0, 0) on the unit square M.

a) Use the transformation in the diagram below to calculate the exact values of the coordinates of A′ and C′



In $\Delta OA'N$, $ON = \cos 45°$ and $A'N = \sin 45°$

$A'(x,y) \rightarrow (\cos 45°, \sin 45°) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

In $\Delta OC'K$, $OK = \cos 45°$ and $C'K = -\sin 45°$

$C'(x,y) \rightarrow (\cos 45°, -\sin 45°) = \cancel{\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)}$

$\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

NOTE: $|OB| = \sqrt{|OA|^2 + |AB|^2}$
$= \sqrt{1^2 + 1^2}$
$= \sqrt{2}$

$B'(x,y) = (0, \sqrt{2})$

✓✓[Draws right triangles to solve the trig values]
✓✓✓[Correct coordinates]

b) Fill in the results to the matrix operation.

$R_{45} \times M = M'$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} O & A & B & C \\ \hline 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} O & A' & B' & C' \\ \hline 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \sqrt{2} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

✓✓[Completes the image matrix from the diagram]

c) Hence complete the following:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

✓[Defines the transformation matrix]

d) Complete the trig ratios

$\sin 45° = \frac{1}{\sqrt{2}}$ $\qquad$ $\cos 45° = \frac{1}{\sqrt{2}}$

and substitute the trig terms in place of the ratios in the transformation matrix.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix}$$

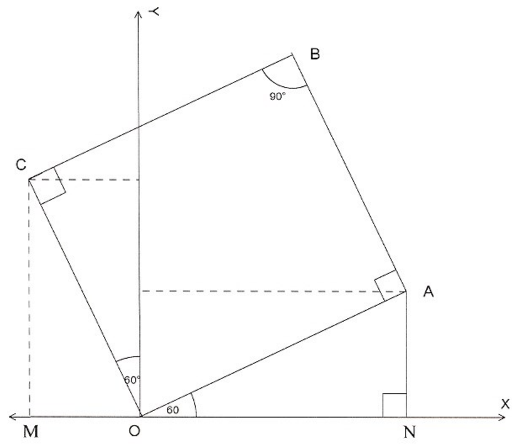
✓[Defines the trig values]

✓[Replaces them in the matrix]

Question 12 [4, 2, 1, 2 = 9 marks]

Suppose matrix R_{60} performs a rotation 60^0 anti-clockwise about the origin (0, 0) on the unit square M.

a) Use the transformation in the diagram below to calculate the exact values of the coordinates of A′ and C′.
<u>You do NOT need to evaluate the coordinates of B′</u>



In ΔOA′N, ON = cos 60° and A′N = sin 60°

A′(x,y) → (cos 60°, sin 60°) = $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

In ΔOC′K, OK = cos 60° and C′K = -sin 60°

C′(x,y) → (cos 60°, -sin 60°) = $\cancel{\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)}$ $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

✓✓[Draws right triangles to solve the trig values]
✓✓[Correct coordinates]

b) Fill in the results to the matrix operation.

$R_{60} \times M = M'$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{array}{c} \begin{array}{cccc} O & A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} O & A' & B' & C' \end{array} \\ \begin{bmatrix} 0 & \frac{1}{2} & x_{B'} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & y_{B'} & \frac{1}{2} \end{bmatrix} \end{array}$$

✓✓[Completes the image matrix from the diagram]

c) Hence complete the following:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

✓[Defines the transformation matrix]

d) Complete the trig ratios

$\sin 60° = \frac{\sqrt{3}}{2}$ $\qquad$ $\cos 60° = \frac{1}{2}$

and substitute the trig terms in place of the ratios in the transformation matrix.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \cos 60° & -\sin 60° \\ \sin 60° & \cos 60° \end{bmatrix}$$

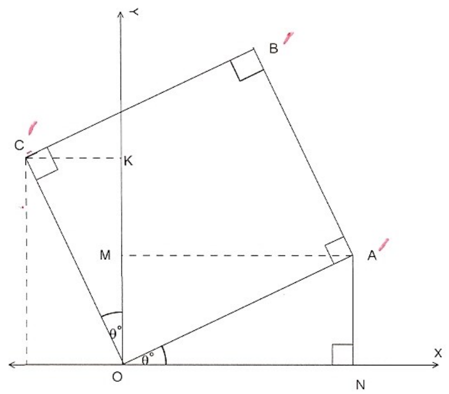
✓[Defines the trig values]

✓[Replaces them in the matrix]

Question 13 [4, 2 = 6 marks]

Suppose matrix R_θ performs a rotation θ^0 anti-clockwise about the origin (0, 0) on the unit square M.

a) Use the transformation in the diagram below to calculate the exact values of the coordinates of A′ and C′.
<u>You do NOT need to evaluate the coordinates of B′</u>



In $\Delta OA'N$, $ON = \cos\theta$ and $A'N = \sin\theta$

$A'(x,y) \rightarrow (\cos\theta, \sin\theta)$

In $\Delta OC'K$, $OK = \cos\theta$ and $C'K = -\sin\theta$

$C'(x,y) \rightarrow \cancel{(\cos\theta, -\sin\theta)}$

$(-\sin\theta, \cos\theta)$

✓✓[Draws right triangles to solve the trig values]
✓✓[Correct coordinates]

b) Hence or otherwise define the transformation matrix for rotation of $\theta°$ anti-clockwise about the origin.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

✓✓[Defines the transformation matrix]